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1.0 PERFORMANCE PREDICTION

1.1 Sonar Equation Application

The sonar equations developed for handling the detection of sonar targets are the basis for predicting the performance of bathymetric systems. The sonar equation most generally used expresses the system performance as a measure of signal-to-noise ratio at the transducer for a specified depth. The applicable equation for bathymetric systems can be written in two forms and the choice of propagation loss model determines which form is used. The general form of the sonar equation used is

\[
N_E = L_S - N_W - L_N - N_{RD} - N_D
\]

where

- \( N_E \) = Signal excess, expressed in decibels (dB) as the ratio of signal to noise at the system input above that required for reliable performance, and is a measure of detection reliability.
- \( L_S \) = Source level, a sound pressure expressed in dB referred to 1 dyne/cm\(^2\) at a distance of 1 yard from the transducer.
- \( N_W \) = Total propagation loss (dB).
- \( L_N \) = System noise level (dB).
- \( N_{RD} \) = Required signal-to-noise ratio for display used (recorder, digitizer) (dB).
- \( N_D \) = Two-way transmission loss in dome or other interface between the transducer and the water (dB).

1.1.1 Source Level (L\(_S\))

The source level is the sound pressure level (on axis) expressed in dB relative to 1 \( \mu \)bar at a distance of 1 yard from the transducer. The source level is determined by:

- The amount of electrical transmitting power applied to the transducer.
- The ability of the transducer to convert electrical energy to acoustical energy in the water (i.e., transducer efficiency).
- The ability of the transducer to concentrate the energy into a beam (i.e., transducer gain).

The above factors can be combined in a logarithmic equation to give the source level.
Equation 1-2 \( L_s = 71.6 + 10 \log P_E + N_{DIT} \cdot E \)

where

\( P_E \) = Electrical transmitting power in watts (average pulse power).
\( E \) = Transducer efficiency expressed in dB \( E = 10 \log \left( \frac{100}{n} \right), \) \( n \) = efficiency in percent
\( N_{DIT} \) = Transducer transmitting directivity index.

71.6 = Parameter conversion constant.

In addition, the directivity index for a circular piston transducer can be calculated with a good approximation by

Equation 1-3 \( N_{DIT} = 10 \log \frac{4 \pi A}{\lambda^2} \)

where
\( A \) = Effective radiating surface area of transducer in square inches
\( \lambda \) = Wavelength in water (inches / cycle)
\( \lambda = \frac{c}{f} \), where \( c \) = velocity of sound in water, \( f \) = frequency

1.1.2 Propagation Loss (\( N_W \))

There are two basic propagation loss models, which can be used in the sonar equation to calculate the depth capability of a bathymetric system. The difference between the two models is based on the assumptions concerning the physics of the reflection from the bottom. The first model considers the bottom echo to be primarily a coherent signal reflection with a bottom loss factor to account for absorption and scattering losses. This model is sometimes referred to as the "lossy mirror" or "specula" model.

The second model assumes no specularly reflected component, and the bottom reflected signal is an incoherent addition of a large number of independent signal components. Each unit element of the illuminated area contributes a small amount of reflected signal power dependent on a back-scattering coefficient. In this case, the reflected signal is a function of the illuminated area. This model is referred to as the target strength or scattering model.
1.1.2.1 Specular Model

The propagation loss is given by

\[ N_W = 20 \log 2R + \frac{2 \alpha R}{1000} + N_{BL} \]  \hspace{1cm} (Equation 1-4)

where

- \( N_W \) = Total propagation loss in decibels
- \( R \) = Depth, yards
- \( \alpha \) = Attenuation coefficient, dB/kyd
- \( N_{BL} \) = Bottom loss caused by absorption and scattering.

Typical values of bottom loss, \( N_{BL} \), at an operating frequency of 12 kHz range from 10 to 30 dB, with value of 20 dB used for an average value. For slope angles less than one half the acoustic beamwidth, there will be a strong specular return and this first model applies.

1.1.2.2 Scattering Model

The propagation loss for this model is

\[ N_W = 40 \log R + \frac{2 \alpha R}{1000} - 10 \log A - S_B \]  \hspace{1cm} (Equation 1-5)

where

- \( S_B \) = Bottom back scattering strength per unit area, a function of bottom slope.
- \( A \) = Effective illuminated area in square yards.

Measured values of back scattering coefficient \( S_B \) are listed in Table 1.

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( S_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(^\circ)</td>
<td>-10</td>
</tr>
<tr>
<td>10(^\circ)</td>
<td>-12</td>
</tr>
<tr>
<td>15(^\circ)</td>
<td>-15</td>
</tr>
</tbody>
</table>

Table 1 Measured Values of Backscattering Coefficient, \( S_B \) as a function of Bottom Slope

There are two methods in calculating the ensonified or illuminated area, depending on the depth.

The first area (A) is a function of the transmitted beamwidth and depth. For a flat bottom, \( \phi = 0\(^\circ\) \) and the area illuminated by a conical beam is determined by the geometry below.
Figure 1 Area illuminated by a conical beam

\[ A = \pi \left( \frac{L}{2} \right)^2 \]

\[ L = R \tan \frac{\theta}{2} \]

Then

Where

\( R \) = depth below transducer.

\[ A = \pi R^2 \tan \frac{\theta}{2} \]
The second area \( (A) \) is a function of the transmitted beamwidth, depth, and pulse duration.

\[
A_2 = \pi r^2 \\

r = \sqrt{R + \frac{T_c}{2}}^2 \cdot R^2 \\

r = \sqrt{R T_c + \frac{T_c}{2}}^2 \\

RT_c \gg \frac{T_c}{2}^2 \\

r = \sqrt{R T_c} \\

A_2 = \pi R T_c 
\]

\( \Theta = \) Acoustic Beamwidth  \\
\( R = \) Depth Below Transducer  \\
\( T = \) Pulse Width  \\
\( c = \) Speed of Sound
\[
\begin{align*}
A_1 &= A_2 \text{ occurs at the cross-over depth.} \\
\pi R^2 \tan \frac{\theta}{2} &= \pi RT_c \\
R_c &= \frac{T_c}{\tan \frac{\theta}{2}} \\
R_c &\text{ is the cross-over depth.}
\end{align*}
\]

For depths \( R \leq R_c \), use \( A_1 \) as illuminated area.

For depths \( R > R_c \), use \( A_2 \) as illuminated area.

For a sloping bottom, the calculation of the ensonified area becomes more complex and requires an analysis of the bathymetric geometry, including bottom slope and transmitted pulse duration.

### 1.1.2.3 Attenuation Coefficient (\( \alpha_0 \))

The final consideration in determining the propagation loss is the determination of the attenuation coefficient (\( \alpha_0 \)) in Equation 1-4. The attenuation coefficient becomes a limiting factor in the propagation of acoustic signals as the frequency increases. The attenuation coefficient becomes significant for frequencies above 10 kHz and is a parameter whose value has been determined by a combination of theory and experimental measurements.

The equation most often used is that fitted by Schulkin and Marsh\(^5\) to some 30,000 measurements made at sea.

**Equation 1-6**

\[
\alpha_0 = A - \left( \frac{S}{f_T} - \frac{B f^2}{f_T^2} \right) \text{ db/kyd}
\]

where

- \( S = \text{Salinity in parts per thousand (typically = 35)} \)
- \( A = \text{Constant = 1.86 x 10^{-2}} \)
- \( B = \text{Constant = 2.68 x 10^{-2}} \)
- \( f = \text{frequency in kHz} \)
- \( f_T = \text{temperature dependent relaxation frequency = 21.9 x 10^2 \left( \frac{6 - 1520}{T + 273} \right)} \)
- \( T = \text{in degrees centigrade (f_T = 72.7 at 40° F)} \)

Some values for \( \alpha \) are given in **Table 2** for typical frequencies used in bathymetric systems.
Table 2: Attenuation Coefficient in dB/kyd ($\delta_{o}$)

<table>
<thead>
<tr>
<th>Temperature</th>
<th>3.5</th>
<th>7</th>
<th>12</th>
<th>15</th>
<th>20</th>
<th>40</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>40°F</td>
<td>0.1141</td>
<td>0.4534</td>
<td>1.310</td>
<td>2.018</td>
<td>3.481</td>
<td>11.60</td>
<td>34.64</td>
<td>56.53</td>
</tr>
<tr>
<td>50°F</td>
<td>0.0889</td>
<td>0.3544</td>
<td>1.031</td>
<td>1.597</td>
<td>2.786</td>
<td>9.898</td>
<td>35.34</td>
<td>61.35</td>
</tr>
<tr>
<td>60°F</td>
<td>0.0700</td>
<td>0.2794</td>
<td>0.8158</td>
<td>1.268</td>
<td>2.277</td>
<td>8.254</td>
<td>35.35</td>
<td>66.14</td>
</tr>
<tr>
<td>70°F</td>
<td>0.0558</td>
<td>0.2230</td>
<td>0.6526</td>
<td>1.016</td>
<td>1.793</td>
<td>6.823</td>
<td>31.95</td>
<td>69.55</td>
</tr>
<tr>
<td>80°F</td>
<td>0.0447</td>
<td>0.1786</td>
<td>0.5236</td>
<td>0.8163</td>
<td>1.444</td>
<td>5.592</td>
<td>28.62</td>
<td>70.69</td>
</tr>
<tr>
<td>90°F</td>
<td>0.0362</td>
<td>0.1448</td>
<td>0.4248</td>
<td>0.6628</td>
<td>1.175</td>
<td>4.598</td>
<td>25.03</td>
<td>69.18</td>
</tr>
</tbody>
</table>

The effect of pressure or depth on the attenuation coefficient is to modify it by

$$\delta_d = \delta_o (1 - 0.965 \times 10^{-5}d)$$

where

- $\delta_d$ = Average attenuation coefficient used in bathymetric equation for depth sounding at depth ($d$)
- $d$ = Depth of water between transducer and bottom in feet
- $\delta_o$ = Value of attenuation coefficient before being corrected for depth.

Some values of $\delta_d$ for an average water temperature of 40°F are given in Table 3.

Table 3: Average Attenuation Coefficient in dB/kyd ($\delta_d$) as a Function of Depth Used for Depth Sounding ($T = 40°F$)

<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>3.5</th>
<th>7</th>
<th>12</th>
<th>15</th>
<th>20</th>
<th>40</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>35,000</td>
<td>0.076</td>
<td>0.299</td>
<td>0.867</td>
<td>2.33</td>
<td>2.30</td>
<td>7.68</td>
<td>22.9</td>
<td>37.4</td>
</tr>
<tr>
<td>30,000</td>
<td>0.081</td>
<td>0.322</td>
<td>0.930</td>
<td>1.43</td>
<td>2.47</td>
<td>8.24</td>
<td>24.6</td>
<td>40.1</td>
</tr>
<tr>
<td>25,000</td>
<td>0.087</td>
<td>0.244</td>
<td>0.994</td>
<td>1.53</td>
<td>2.64</td>
<td>8.80</td>
<td>26.3</td>
<td>42.9</td>
</tr>
<tr>
<td>20,000</td>
<td>0.092</td>
<td>0.365</td>
<td>1.05</td>
<td>1.62</td>
<td>2.81</td>
<td>9.36</td>
<td>27.9</td>
<td>45.6</td>
</tr>
<tr>
<td>18,000</td>
<td>0.094</td>
<td>0.374</td>
<td>1.08</td>
<td>1.67</td>
<td>2.87</td>
<td>9.58</td>
<td>28.6</td>
<td>46.7</td>
</tr>
<tr>
<td>15,000</td>
<td>0.097</td>
<td>0.387</td>
<td>1.12</td>
<td>1.72</td>
<td>2.98</td>
<td>9.92</td>
<td>29.6</td>
<td>48.3</td>
</tr>
<tr>
<td>12,000</td>
<td>0.101</td>
<td>0.400</td>
<td>1.16</td>
<td>1.78</td>
<td>3.08</td>
<td>10.2</td>
<td>30.6</td>
<td>50.0</td>
</tr>
<tr>
<td>10,000</td>
<td>0.103</td>
<td>0.409</td>
<td>1.18</td>
<td>1.82</td>
<td>3.14</td>
<td>10.5</td>
<td>31.3</td>
<td>51.0</td>
</tr>
<tr>
<td>8,000</td>
<td>0.105</td>
<td>0.418</td>
<td>1.21</td>
<td>1.86</td>
<td>3.21</td>
<td>10.7</td>
<td>32.0</td>
<td>52.2</td>
</tr>
<tr>
<td>5,000</td>
<td>0.109</td>
<td>0.431</td>
<td>1.25</td>
<td>1.92</td>
<td>3.31</td>
<td>11.0</td>
<td>33.0</td>
<td>53.8</td>
</tr>
<tr>
<td>2,000</td>
<td>0.112</td>
<td>0.444</td>
<td>1.28</td>
<td>1.98</td>
<td>3.41</td>
<td>11.4</td>
<td>34.0</td>
<td>55.4</td>
</tr>
<tr>
<td>1,000</td>
<td>0.115</td>
<td>0.448</td>
<td>1.30</td>
<td>2.00</td>
<td>3.45</td>
<td>11.5</td>
<td>33.4</td>
<td>56.0</td>
</tr>
<tr>
<td>500</td>
<td>0.114</td>
<td>0.451</td>
<td>1.30</td>
<td>2.01</td>
<td>3.46</td>
<td>11.5</td>
<td>34.5</td>
<td>56.2</td>
</tr>
<tr>
<td>0</td>
<td>0.114</td>
<td>0.453</td>
<td>1.31</td>
<td>2.02</td>
<td>3.48</td>
<td>11.6</td>
<td>34.6</td>
<td>56.5</td>
</tr>
</tbody>
</table>
1.1.3 System Noise Level ($L_N$)

System-noise level includes self noise generated by the vessel, ambient noise that exists in the water, and reverberation noise which is a function of the propagation paths and configuration of the acoustic system.

An appropriate value of self noise generated by the vessel should be determined for the installation considered. It is usually the self-noise level that limits bathymetric system performance. Self noise is expressed as a spectrum level and specified in dB/1 bar in a 1 Hz band. It then becomes necessary to increase the spectrum level by a bandwidth correction factor ($N_{BW}$) determined by the bandwidth of the receiving system. The bandwidth correction factor is expressed in dB as:

$$N_{BW} = 10 \log \frac{BW}{1\text{Hz}}$$

where

$N_{BW} = \text{bandwidth correction factor}$
$BW = \text{bandwidth of receiving system in Hz.}$

Self-noise levels are generally assumed as isotropic sources. This assumes that noise received at the hydrophone has no directional characteristics, and consequently, the directivity of the transducer will act to reduce the self noise that passes into the receiving system. The total noise term in the sonar equation is then expressed by:

$$L_N = L_a + N_{BW} + N_{DIR}$$

where

$L_N = \text{Total self noise in the receiver bandwidth}$
$L_a = \text{Spectrum level of self noise in a 1 Hz bandwidth}$
$N_{BW} = \text{Bandwidth correction factor}$
$N_{DIR} = \text{Directivity index of receiving transducer.}$

It should be remembered that self noise, like ambient noise, is not always isotropic in its directional characteristics. Consequently, the use of the directivity index as an exact reduction of the noise level entering the receiving system is not always satisfactory. For example, if a highly directional noise source exists within the beamwidth of the receiving system, then the directivity index will be almost meaningless as a measure of discrimination against noise.

Contributions of the electronics in a bathymetric system to the self-noise level usually occurs only at high frequencies; potential electrical noise sources in the receiving system should be reduced to a value well below the limiting ambient acoustic noise level.
Ambient noise is that background noise level that exists in the ocean due mainly to marine life and man-made noise. Man-made noise will vary in its origin e.g., for surveys in a harbor, man-made noise may be predominately originated on land. Whereas, for bathymetric operations near shipping lanes, man-made noise will be the aggregate of nearby and distant shipping noise.

The lower limit of ambient noise is that generated by the movement of the water itself. Water noise may be generated in a number of ways, again depending on where the bathymetric operations are being performed. Measurements of water noise in a wide variety of locations, and under varying conditions, have yielded statistical estimates as to its magnitude. As might be expected, the resulting averages have been found to vary with the state of the sea. Values for the spectrum level of water noise as a function of frequency and sea state are shown in Figure 1. The significance of the noise levels shown in Figure 1 is that the dominant noise level (self noise) that is present during bathymetric operations is often given relative to the spectrum levels shown. Table 4-4 presents the sea-state numbers, and a description of the conditions they represent.

Another source of noise in bathymetric systems is that caused by reverberation. Reverberation can be defined as the re-radiations of sound by the sum of all the unwanted scattering contributions from all scatterers not pertinent to the bathymetric measurement. The reverberation that occurs can come from the water surface, re-radiation from a body of marine life scatterers (volume reverberation) in the sea, or from the bottom. Bottom reverberation is a significant noise source consideration in active sonar systems. For bathymetric applications, bottom reverberation is considered only when the application is to discriminate some feature that extends above or is buried in the bottom and might be obscured.

Figure 3. Deep Water Ambient Noise Levels
Table 4. *State of the Sea*

<table>
<thead>
<tr>
<th>Sea State Number</th>
<th>Description</th>
<th>Height of Waves (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Calm Sea</td>
<td>0-1</td>
</tr>
<tr>
<td>1</td>
<td>Smooth Sea</td>
<td>1-2</td>
</tr>
<tr>
<td>2</td>
<td>Slight Waves</td>
<td>2-3</td>
</tr>
<tr>
<td>3</td>
<td>Moderate Sea</td>
<td>3-5</td>
</tr>
<tr>
<td>4</td>
<td>Rough Sea</td>
<td>5-8</td>
</tr>
<tr>
<td>5</td>
<td>Very Rough Sea</td>
<td>8-12</td>
</tr>
<tr>
<td>6</td>
<td>High Sea</td>
<td>12-20</td>
</tr>
<tr>
<td>7</td>
<td>Very High Sea</td>
<td>20-40</td>
</tr>
<tr>
<td>8</td>
<td>Precipitous Sea</td>
<td>40+</td>
</tr>
<tr>
<td>9</td>
<td>Confused Sea</td>
<td>---</td>
</tr>
</tbody>
</table>

1.1.4 **Recognition Differential (**$N_{RD}$** )**

An acceptable value of recognition differential has been arrived at empirically by many observers using various types of displays in bathymetric applications. The two principal display techniques for bathymetric systems are the dry paper precision recorder and digital. Current design practice dictates that when a signal-to-noise ratio of 10 dB is used for the recognition differential for a dry paper recorder, and a signal-to-noise ratio of 14 dB is used for digital systems, reliable data recognition is achieved by the system.

1.1.5 **Acoustic Interface Loss (**$N_D$** )**

When the transducer transmits in any manner other than directly into the sea, or final propagation medium, then an acoustic interface loss (two-way transmission loss) must be considered when using the general form of the sonar equation. The technique for calculating this loss is explained in detail in Section 3.2.3.